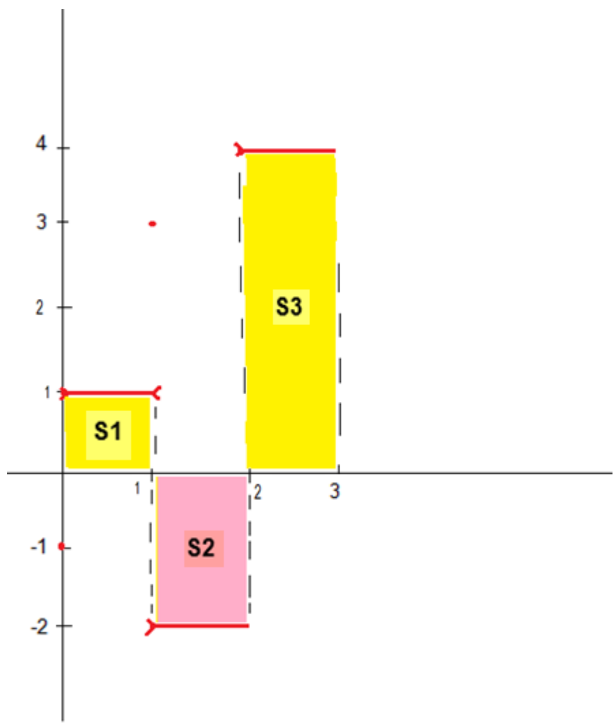
**Ecole Nationale Polytechnique de Constantine**

**Première Année Classe Préparatoire**

# Corrigé - Série N°7 Analyse 2

**Exercice 1.** *La fonction $f$ dfinie sur $[0,3]$ valeurs dans $\mathbb{R}$ est dfinie par*

$$f(x) = \begin{cases} -1 \ si \ x = 0, \\ 1 \ si \ 0 < x < 1, \\ 3 \ si \ x = 1, \\ -2 \ si \ 1 < x \leq 2, \\ 4 \ si \ 2 < x \leq 3. \end{cases}$$




(1) *L'intégrale de la fonction en escalier $f$ sur l'intervalle $[0,3]$ est :*
$\int_0^3 f(t)\,dt = S1 - S2 + S3 = 1(1-0) - 2(2-1) + 4(3-2) = 1 - 2 + 4 = 3.$

(2) *Posons $F(x) = \int_0^x f(t)\,dt$, explicitons $F$ :*

- *Si* $x = 0,\ F(0) = \int_0^0 f(t)\,dt = 0.$
- *Si* $x \in ]0,1[,\ F(x) = \int_0^x 1\,dt = 1(x-0) = x.$
- *Si* $x = 1,\ F(1) = \int_0^1 f(t)\,dt = 1(1-0) = 1.$ *Donc*
$$F(x) = x \ pour \ 0 \leq x \leq 1.$$
- *Si* $x \in ]1,2],\ F(x) = 1.(1-0) + (-2)(x-1) = -2x + 3.$
- *Si* $x \in ]2,3],\ F(x) = 1.(1-0) + (-2)(2-1) + 4(x-2) = 4x - 9.$

*Alors*

$$F(x) = \begin{cases} x \;\; si \; 0 \leq x \leq 1, \\ -2x+3 \;\; si \; 1 < x \leq 2, \\ 4x-9 \;\; si \; 2 < x \leq 3. \end{cases}$$

(3) *Comme $f$ est une fonction en escalier alors elle est intégrable, d'après le théorème fondamentale de l'analyse $F$ est continue.*

(4) *Il est facile de vérifier que $F$ n'est pas dérivable en $x = 2$ alors elle ne l'est pas sur $[0, 3]$.*

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**Exercice 2.**

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**Exercice 3.** *Procédons par l'absurde, supposons que $\int\limits_a^b f(t)\,dt = 0$ et $f \not\equiv 0$ sur $[a, b]$ donc*

$$\exists\, x_0 \in [a, b] : f(x_0) \neq 0 \;(f(x_0) > 0).$$

*La fonction $f$ est continue en $x_0 \in [a, b]$ alors*

$$\exists\, \delta > 0, \forall x \in [x_0 - \delta, x_0 + \delta] : |f(x_0) - f(x)| \leq \varepsilon$$

*On prend $\varepsilon = \frac{f(x_0)}{2} > 0$,*

$$\exists \delta > 0, \forall x \in [x_0 - \delta, x_0 + \delta] : -\varepsilon = -\frac{f(x_0)}{2} \leq f(x_0) - f(x)$$

*donc $0 < \frac{f(x_0)}{2} \leq f(x)$, et on a*

$$\int\limits_a^b f(t)\,dt = \int\limits_a^{x_0-\delta} f(t)\,dt + \int\limits_{x_0-\delta}^{x_0+\delta} f(t)\,dt + \int\limits_{x_0+\delta}^{b} f(t)\,dt.$$

*Comme $f$ est positive alors*

$$\int\limits_a^{x_0-\delta} f(t)\,dt \geq 0 \;\; et \int\limits_{x_0+\delta}^{b} f(t)\,dt \geq 0.$$

*On a $f(x) \geq \frac{f(x_0)}{2}$ alors*

$$\int\limits_{x_0-\delta}^{x_0+\delta} f(t)\,dt \geq \int\limits_{x_0-\delta}^{x_0+\delta} f(x_0)\,dt = \delta f(x_0).$$

*Finalement $\int\limits_a^b f(t)\,dt \geq \delta f(x_0) > 0$ donc $0 < \int\limits_a^b f(t)\,dt = 0$. Absurde.*

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**Exercice 4.** (1) *Pour $x > 0$ on a $\frac{x^n}{1+x} \leq x^n$, donc*

$$I_n \leq \int\limits_0^1 x^n\,dx = \left[\frac{1}{n+1} x^{n+1}\right]_0^1 = \frac{1}{n+1}.$$

*Donc $I_n \to 0$ lorsque $n \to +\infty$.*

(2) $I_n + I_{n+1} = \int\limits_0^1 \frac{x^n + x^{n+1}}{1+x}\,dx = \int\limits_0^1 x^n \frac{1+x}{1+x}\,dx = \int\limits_0^1 x^n\,dx = \frac{1}{n+1}.$

(3) *Soit* $S_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{(-1)^n}{n+1} = \sum\limits_{k=0}^{n} \frac{(-1)^k}{k+1}$. *Par la question précédente on a*

$$\begin{aligned} S_n = \sum_{k=0}^{n} \frac{(-1)^k}{k+1} &= \sum_{k=0}^{n} (-1)^k (I_k + I_{k+1}) \\ &= (I_0 + I_1) - (I_1 + I_2) + (I_2 + I_3) - (I_3 + I_4) + ... + (-1)^n (I_n + I_{n+1}) \\ &= I_0 + (-1)^n I_{n+1}. \end{aligned}$$

*Comme* $I_{n+1} \to 0$ *quand* $n \to +\infty$, $\lim\limits_{n\to+\infty} S_n = I_0 + \lim\limits_{n\to+\infty} (-1)^n I_{n+1} = I_0$.

*Mais* $I_0 = \int\limits_0^1 \frac{1}{(1+x)}\, dx = [\ln(1+x)]_0^1 = \ln 2$.

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**Exercice 5.** (1) *Calculons* $\lim\limits_{n\to+\infty} U_n$,

$$\begin{aligned} U_n &= \tfrac{1}{n} \left(\sin(\tfrac{\pi}{n}) + \sin(\tfrac{2\pi}{n}) + ... + \sin(\tfrac{n\pi}{n})\right) \\ &= \tfrac{1-0}{n} \left(\sin\pi\left(0 + \tfrac{1.(1-0)}{n}\right) + \sin\pi\left(0 + \tfrac{2.(1-0)}{n}\right) + ... + \sin\pi\left(0 + \tfrac{n.(1-0)n}{n}\right)\right) \\ &= \sum_{k=1}^{n} \tfrac{1-0}{n} \sin\pi\left(0 + \tfrac{k.(1-0)}{n}\right), \end{aligned}$$

*avec* $f(x) = \sin\pi x$ *et* $[a, b] = [0, 1]$.

*La fonction* $f$ *est continue sur* $[0, 1]$ *donc intégrable et on a :*

$$\lim_{n\to+\infty} U_n = \int\limits_0^1 \sin(\pi x)\, dx = \left[-\frac{1}{\pi}\cos(\pi x)\right]_0^1 = \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}.$$

(2) *Calculons* $\lim\limits_{n\to+\infty} V_n$,

$$\begin{aligned} V_n &= n\left(\tfrac{1}{(n+1)^2} + \tfrac{1}{(n+2)^2} + ... + \tfrac{1}{(n+n)^2}\right) \\ &= n\left(\tfrac{1}{\left(n\left(1+\frac{1}{n}\right)\right)^2} + \tfrac{1}{\left(n\left(1+\frac{2}{n}\right)\right)^2} + ... + \tfrac{1}{(n(1+1))^2}\right) \\ &= n\left(\tfrac{1}{n^2\left(1+\frac{1}{n}\right)^2} + \tfrac{1}{n^2\left(1+\frac{2}{n}\right)^2} + ... + \tfrac{1}{n^2(1+1)^2}\right) \\ &= \tfrac{n}{n^2}\left(\tfrac{1}{\left(1+\frac{1}{n}\right)^2} + \tfrac{1}{\left(1+\frac{2}{n}\right)^2} + ... + \tfrac{1}{(1+1)^2}\right) \\ &= \tfrac{1-0}{n}\left(\sum_{k=1}^{n} \tfrac{1}{\left(1+\frac{k}{n}\right)^2}\right) \end{aligned}$$

*avec* $f(x) = \frac{1}{(1+x)^2}$ *et* $[a, b] = [0, 1]$.

*La fonction* $f$ *est continue sur* $[0, 1]$ *donc intégrable et on a :*

$$\lim_{n\to+\infty} V_n = \int\limits_0^1 \frac{1}{(1+x)^2}\, dx = \left[-\frac{1}{1+x}\right]_0^1 = \frac{1}{2}.$$

(3) *Calculons* $\lim\limits_{n\to+\infty} W_n$,

$$\begin{aligned} W_n = \tfrac{\sqrt{1}+\sqrt{2}+...+\sqrt{n}}{n\sqrt{n}} &= \tfrac{1-0}{n}\left(\tfrac{\sqrt{1}+\sqrt{2}+...+\sqrt{n}}{\sqrt{n}}\right) \\ &= \tfrac{1-0}{n}\left(\tfrac{\sqrt{1}}{\sqrt{n}} + \tfrac{\sqrt{2}}{\sqrt{n}} + ... + \tfrac{\sqrt{n}}{\sqrt{n}}\right) \\ &= \tfrac{1-0}{n}\left(\sqrt{\tfrac{1}{n}} + \sqrt{\tfrac{2}{n}} + ... + \sqrt{\tfrac{n}{n}}\right) \\ &= \tfrac{1-0}{n}\left(\sqrt{\tfrac{1}{n}} + \sqrt{\tfrac{2}{n}} + ... + \sqrt{\tfrac{n}{n}}\right) \\ &= \tfrac{1-0}{n}\left(\sqrt{\tfrac{0+1.(1-0)}{n}} + \sqrt{\tfrac{0+2.(1-0)}{n}} + ... + \sqrt{\tfrac{0+n.(1-0)}{n}}\right) \\ &= \tfrac{1-0}{n}\sum_{k=1}^{n}\sqrt{\tfrac{k}{n}} \end{aligned}$$

*avec* $f(x) = \sqrt{x}$ *et* $[a, b] = [0, 1]$.

*La fonction* $f$ *est continue sur* $[0, 1]$ *donc intégrable et on a :*

$$\lim_{n \to +\infty} W_n = \int_0^1 \sqrt{x}\, dx = \left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}.$$

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## École Polytechnique de Constantine

## TD $N^o 7$ Analyse II

**Exercice 1.**
*énoncé de l'exercice.1*

**Solution.**
*solution de l'exercice.1*

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**Exercice 2.**
*énoncé de l'exercice.2*

**Solution.**
*solution de l'exercice.2*

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**Exercice 3.**
*énoncé de l'exercice.3*

**Solution.**
*solution de l'exercice.3*

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**Exercice 4.**
*énoncé de l'exercice.4*

**Solution.**
*solution de l'exercice.4*

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**Exercice 5.**
*énoncé de l'exercice.5*

**Solution.**
*solution de l'exercice.5*

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**Exercice 6.**
*Calculer* $\int_0^1 t^2 dt$ *en utilisant les sommes de Riemann.*

**Solution.**
*Rappelons que, si la fonction* $f : [a, b] \to \mathbb{R}$ *est continue alors*

$$\int_a^b f(t)dt = \lim_{n\to+\infty} \frac{b-a}{n} \sum_{k=1}^{n} f(a + k\frac{b-a}{n}).$$

*On pose* $a = 0$, $b = 1$ *et* $f(t) = t^2$, *on obtient*

$$\begin{aligned}\int_0^1 t^2 dt &= \lim_{n\to+\infty} \frac{1}{n} \sum_{k=1}^{n} (\frac{k}{n})^2 = \lim_{n\to+\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2 \\ &= \lim_{n\to+\infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6} = \lim_{n\to+\infty} \frac{n(n+1)(2n+1)}{6n^3} \\ &= \lim_{n\to+\infty} \frac{2n^3}{6n^3} = \frac{1}{3}\end{aligned}$$

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**Exercice 7.**
*Décomposer la fraction rationnelle suivante en éléments simples dans* $\mathbb{C}$ *puis dans* $\mathbb{R}$.

$$\frac{1}{x(x^2+1)^2}$$

**Solution.**
***1.*** *Décomposition dans* $\mathbb{C}$ *: on écrit* $x^2 + 1 = (x - i)(x + i)$, *d'où*

$$\frac{1}{x(x^2+1)^2} = \frac{1}{x(x-i)^2(x+i)^2} = \frac{a}{x} + \frac{b}{x-i} + \frac{c}{(x-i)^2} + \frac{d}{x+i} + \frac{e}{(x+i)^2}$$

*on calcule* $a$, $c$, *et* $e$ *en utilisant la méthode multiplicative :*

$$a = \frac{1}{(x^2+1)^2} - \frac{bx}{x-i} - \frac{cx}{(x-i)^2} - \frac{dx}{x+i} - \frac{ex}{(x+i)^2}, \quad \textit{posons} \quad x = 0 \Rightarrow a = 1$$

$$c = \frac{1}{x(x+i)^2} - \frac{a(x-i)^2}{x} - b(x-i) - \frac{d(x-i)^2}{x+i} - \frac{e(x-i)^2}{(x+i)^2}, \quad \textit{posons} \quad x = i \Rightarrow c = \frac{i}{4}$$

$$e = \frac{1}{x(x-i)^2} - \frac{a(x+i)^2}{x} - \frac{b(x+i)^2}{x-i} - \frac{c(x+i)^2}{(x-i)^2} - d(x+i), \quad \textit{posons} \quad x = -i \Rightarrow e = \frac{-i}{4}$$

*on obtient*

$$\frac{1}{x(x^2+1)^2} = \frac{1}{x(x-i)^2(x+i)^2} = \frac{1}{x} + \frac{b}{x-i} + \frac{i}{4(x-i)^2} + \frac{d}{x+i} + \frac{-i}{4(x+i)^2}$$

*on calcule* $b$ *et* $d$ *en substituant des valeurs de* $x$ *dans la dernière équation :*

$$\textit{pour} \quad x = 2i \textit{ on obtient } 3b + d = -2$$

$$\textit{pour} \quad x = -2i \textit{ on obtient } -b - 3d = 2$$

*alors :* $b = d = \frac{-1}{2}$ *on conclue*

$$\frac{1}{x(x^2+1)^2} = \frac{1}{x} + \frac{-1}{2(x-i)} + \frac{i}{4(x-i)^2} + \frac{-1}{2(x+i)} + \frac{-i}{4(x+i)^2}$$

*2. Décomposition dans* $\mathbb{R}$

$$\frac{1}{x(x^2+1)^2} = \frac{a}{x} + \frac{bx+c}{(x^2+1)} + \frac{dx+e}{(x^2+1)^2}$$

$$a = \frac{1}{(x^2+1)^2} - \frac{(bx+c)x}{(x^2+1)} + \frac{(dx+e)x}{(x^2+1)^2} \text{ posons } x = 0 \Rightarrow a = 1$$

$$dx + e = \frac{1}{x} - \frac{a(x^2+1)^2}{x} + (bx+c)(x^2+1) \text{ posons } x = i \Rightarrow di + e = -i \Rightarrow d = -1 \text{ et } e = 0$$

*d'où*

$$\frac{1}{x(x^2+1)^2} = \frac{1}{x} + \frac{bx+c}{(x^2+1)} + \frac{-x}{(x^2+1)^2}$$

$$\text{pour } x = 1 \text{ on obtient } b + c = -1$$

$$\text{pour } x = -1 \text{ on obtient } -b + c = 1$$

*Alors* $b = -1$ *et* $c = 0$, *et*

$$\frac{1}{x(x^2+1)^2} = \frac{1}{x} + \frac{-x}{(x^2+1)} + \frac{-x}{(x^2+1)^2}$$

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**Exercice 8.**

*Décomposer les fractions rationnelles suivantes en éléments simples*

$$\frac{x+1}{x(x^2-9)^2} \; ; \; \frac{2x+1}{x^2+9}.$$

**Solution.**

***1.***

$$\frac{x+1}{x(x^2-9)^2} = \frac{x+1}{x(x-3)^2(x+3)^2} = \frac{a}{x} + \frac{b}{x-3} + \frac{c}{(x-3)^2} + \frac{d}{x+3} + \frac{e}{(x+3)^2}$$

$$a = \frac{x+1}{(x^2-9)^2} - \frac{bx}{x-3} - \frac{cx}{(x-3)^2} - \frac{dx}{x+3} - \frac{ex}{(x+3)^2} \overset{x=0}{\Rightarrow} a = \frac{1}{81}$$

$$c = \frac{x+1}{x(x+3)^2} - \frac{a(x-3)^2}{x} - b(x-3) - \frac{d(x-3)^2}{x+3} - \frac{e(x-3)^2}{(x+3)^2} \overset{x=3}{\Rightarrow} c = \frac{1}{27}$$

$$e = \frac{x+1}{x(x-3)^2} - \frac{a(x+3)^2}{x} - \frac{b(x+3)^2}{x-3} - \frac{c(x+3)^2}{(x-3)^2} - d(x+3) \overset{x=-3}{\Rightarrow} c = \frac{1}{54}$$

*alors*

$$\frac{x+1}{x(x-3)^2(x+3)^2} = \frac{1}{81x} + \frac{b}{x-3} + \frac{1}{27(x-3)^2} + \frac{d}{x+3} + \frac{1}{54(x+3)^2}$$

*de plus*

$$\lim_{x\to+\infty} x\frac{x+1}{x(x^2-9)^2} = \lim_{x\to+\infty} (\frac{1}{81} + \frac{bx}{x-3} + \frac{x}{27(x-3)^2} + \frac{dx}{x+3} + \frac{x}{54(x+3)^2})$$

*d'où* $b + d = \frac{-1}{81}$, *et pour* $x = -1$ *on obtient* $-b + 2d = \frac{7}{324}$, *donne*

$$b = \frac{-5}{324} \; ; \; d = \frac{1}{324}$$

*alors*

$$\frac{x+1}{x(x^2-9)} = \frac{1}{81x} + \frac{-5}{324(x-3)} + \frac{1}{27(x-3)^2} + \frac{d}{324(x+3)} + \frac{1}{54(x+3)^2}$$

***2.***

$$\frac{2x+1}{x^2+9} = \frac{2x+1}{(x-3i)(x+3i)} = \frac{a}{x-3i} + \frac{b}{x+3i}$$

$$a = \frac{2x+1}{(x+3i)} - \frac{b(x-3i)}{x+3i} \overset{x=3i}{\Rightarrow} a = \frac{6-i}{6}$$

$$b = \frac{2x+1}{(x-3i)} - \frac{a(x+3i)}{x-3i} \overset{x=-3i}{\Rightarrow} b = \frac{6+i}{6}$$

*et*

$$\frac{2x+1}{x^2+9} = \frac{6-i}{6(x-3i)} + \frac{6+i}{6(x+3i)}$$

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**Exercice 9.**
*Calculer les primitives suivantes*

$$1. \int_0^1 \frac{dx}{x^2+x+1} \quad 2. \int \frac{cos(x)}{cos^2(x)+1}dx \ \ (u = sin(x)) \quad 3. \int \frac{sin(x)}{sin^2(x)+1}dx \ \ (u = cos(x))$$

$$4. \int \frac{dx}{cos^3(x)} \ \ (u = sin(x)) \quad 5. \int \frac{dx}{cos(x)+3} \ \ (u = tan(x))$$

**Solution.**
*1.* $\int_0^1 \frac{dx}{x^2+x+1}$ *; on écrit* $x^2+x+1 = (x+\frac{1}{2})^2 + \frac{3}{4}$, *donc*

$$\int_0^1 \frac{dx}{x^2+x+1} = \int_0^1 \frac{dx}{(x+\frac{1}{2})^2+\frac{3}{4}} = \frac{4}{3}\int_0^1 \frac{dx}{(\frac{2x+1}{\sqrt{3}})^2+1}$$

*on pose* $u = \frac{2x+1}{\sqrt{3}} \Rightarrow du = \frac{2}{\sqrt{3}}$ *, et* $x = 0 \Rightarrow u = \frac{1}{\sqrt{3}}$, $x = 1 \Rightarrow u = \sqrt{3}$. *donc*

$$\begin{aligned}\int_0^1 \frac{dx}{x^2+x+1} &= \frac{4}{3}\frac{\sqrt{3}}{2}\int_0^1 \frac{du}{u^2+1} = \frac{2}{\sqrt{3}}Arctan(u)|_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \\ &= \frac{2}{\sqrt{3}}(Arctan(\sqrt{3}) - Arctan(\frac{1}{\sqrt{3}})) = \frac{2}{\sqrt{3}}(\frac{\pi}{3} - \frac{\pi}{6}) = \frac{\pi}{3\sqrt{3}}\end{aligned}$$

*2.*$\int \frac{cos(x)}{1+cos^2(x)}dx$ *; on pose* $u = sin(x)$ *alors* $du = cos(x)dx$ *et*

$$\int \frac{cos(x)}{1+cos^2(x)}dx = \int \frac{du}{1+(1-u^2)} = \int \frac{du}{2-u^2} = \int \frac{du}{(\sqrt{2}-u)(\sqrt{2}+u)}$$

*de plus*

$$\frac{du}{(\sqrt{2}-u)(\sqrt{2}+u)} = \frac{a}{\sqrt{2}-u} + \frac{b}{\sqrt{2}+u} = \frac{1}{2\sqrt{2}(\sqrt{2}-u)} + \frac{1}{2\sqrt{2}(\sqrt{2}+u)}$$

*donc*

$$\begin{aligned}\int \frac{cos(x)}{1+cos^2(x)}dx &= \int \frac{du}{2\sqrt{2}(\sqrt{2}-u)} + \int \frac{du}{2\sqrt{2}(\sqrt{2}+u)} \\ &= \frac{1}{2\sqrt{2}}(ln|\sqrt{2}+u| - ln|\sqrt{2}-u|) + c \\ &= \frac{1}{2\sqrt{2}}(ln|\sqrt{2}+sin(x)| - ln|\sqrt{2}-sin(x)|) + c \\ &= \frac{1}{2\sqrt{2}}ln|\frac{\sqrt{2}+sin(x)}{\sqrt{2}-sin(x)}| + c\end{aligned}$$

*3.* $\int \frac{sin(x)}{1+sin^2(x)}dx$ *;* $u = cos(x)$ *donne* $du = -sin(x)dx$*, d'après.2 on obtient*

$$\int \frac{sin(x)}{1+sin^2(x)}dx = -\int \frac{du}{2-u^2} = -\frac{1}{2\sqrt{2}}ln|\frac{\sqrt{2}+cos(x)}{\sqrt{2}-cos(x)}| + c$$

*4.* $\int \frac{dx}{cos^3(x)}$ ; $\quad u = sin(x) \Rightarrow du = cos(x)dx$, *alors*

$$\int \frac{dx}{cos^3(x)} = \int \frac{cos(x)dx}{cos^4(x)} = \int \frac{du}{(1-u)^2(1+u)^2}$$
$$= a\int \frac{du}{1-u} + b\int \frac{du}{(1-u)^2} + c\int \frac{du}{1+u} + d\int \frac{du}{(1+u)^2}$$

*ou*

$$a = b = c = d = \frac{1}{4}$$

*donc*

$$\int \frac{dx}{cos^3(x)} = \frac{1}{4}(-ln|1-u| + ln|1+u| - \frac{1}{1-u} + \frac{1}{1+u}) + c$$

5. $\int \frac{dx}{3+cos(x)}$, $\quad u = tan(\frac{x}{2})$, *on a*

$$cos(x) = cos(2\frac{x}{2}) = cos^2(\frac{x}{2}) - sin^2(\frac{x}{2})$$
$$= \frac{cos^2(\frac{x}{2}) - sin^2(\frac{x}{2})}{1} = \frac{cos^2(\frac{x}{2}) - sin^2(\frac{x}{2})}{cos^2(\frac{x}{2}) + sin^2(\frac{x}{2})}$$
$$= \frac{1 - tan^2(\frac{x}{2})}{1 - tan^2(\frac{x}{2})} = \frac{1-u^2}{1+u^2}$$

*et*

$$du = \frac{dx}{2cos^2(\frac{x}{2})} \Rightarrow dx = 2cos^2(\frac{x}{2})du$$

*et comme*

$$cos^2(\frac{x}{2}) = \frac{cos^2(\frac{x}{2})}{cos^2(\frac{x}{2}) + sin^2(\frac{x}{2})} = \frac{1}{1 + tan^2(\frac{x}{2})} = \frac{1}{1+u^2}$$

*donc*

$$dx = \frac{2du}{1+u^2}$$

*alors*

$$\int \frac{dx}{3+cos(x)} = \int \frac{\frac{2du}{1+u^2}}{3 + \frac{1-u^2}{1+u^2}} = \int \frac{du}{2-u^2}$$
$$= \int \frac{du}{(\sqrt{2}-u)(\sqrt{2}-u)} = \int \frac{du}{(\sqrt{2}-u)(\sqrt{2}-u)} = \int \frac{du}{2\sqrt{2}(\sqrt{2}-u)} + \int \frac{du}{2\sqrt{2}(\sqrt{2}-u)}$$
$$= -\frac{ln|\sqrt{2}-u|}{2\sqrt{2}} + \frac{ln|\sqrt{2}+u|}{2\sqrt{2}} + c$$
$$= -\frac{ln|\sqrt{2} - tan(\frac{x}{2})|}{2\sqrt{2}} + \frac{ln|\sqrt{2} + tan(\frac{x}{2})|}{2\sqrt{2}} + c$$

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**Exercice 10.**
*Calculer les primitives des fonctions suivantes*

$$\frac{x}{1+\sqrt{x+1}} \;,\; \frac{1-\sqrt{x}}{1+\sqrt{x}}$$

**Solution.**
*1. On pose* $\sqrt{x+1}=u$*, donc* $u^2-1=x$ *et*

$$\frac{1}{2\sqrt{x+1}}dx=du\Rightarrow dx=2\sqrt{x+1}du=2udu$$

*donc*

$$\begin{aligned}\int\frac{x}{1+\sqrt{x+1}}dx&=\int\frac{(u^2-1)2u}{1+u}du\\&=\int(u-1)(2u)du\\&=2\frac{u^3}{3}-u^2+c\\&=\frac{2}{3}(\sqrt{x+1})^3-(\sqrt{x+1})^2+c\\&=\frac{2}{3}(x+1)^{\frac{3}{2}}-(x+1)+c\end{aligned}$$

*2. On pose* $\sqrt{x}=u\Rightarrow\frac{1}{2\sqrt{x}}dx=du\Rightarrow dx=2u\,du$*, d'où*

$$\begin{aligned}\int\frac{1-\sqrt{x}}{1+\sqrt{x}}dx=\int\frac{1-u}{1+u}2u\,du&\overset{D.U}{=}\int(4-2u-\frac{4}{u+1})du\\&=4u-u^2-4\,ln|u+1|+c\\&=4\sqrt{x}-x-4\,ln|\sqrt{x}+1|+c\end{aligned}$$